Hierarchical hyperbolicity of graph braid groups

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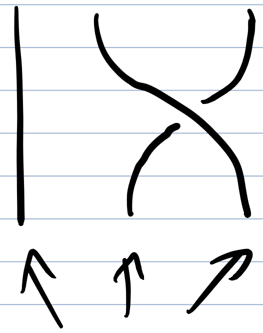
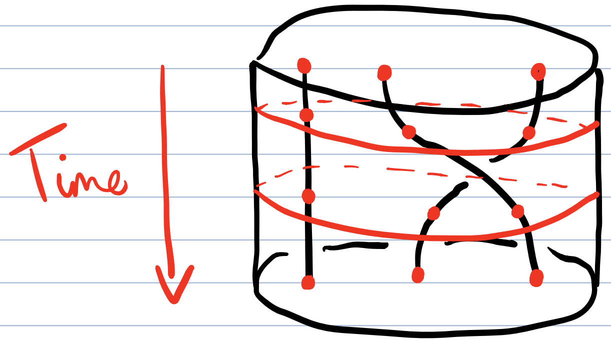
Braids:



A 3-braid

n strands intertwined



Consider this embedded in $D^2 \times I$

An n-braid is n particles moving around D^2 without collision and returning to their start locations.

Can replace the base space D^2 with other spaces X

Configuration space:

$$C_n(X) = \{(x_1, \dots, x_n) \in X^n \mid x_i \neq x_j \ \forall i \neq j\}$$

Loop in $C_n(X) \rightsquigarrow$ n-braid

$$B_n(X) = \pi_1\big(\underbrace{C_n(X)/S_n}_{UC_n(X)}\big)$$

Braid group

<u>Theorem</u>: (Abrams, Crisp-Wiest) If X is a finite graph Γ, then $UC_n(X)$ deformation retracts onto a compact special cube complex.

<u>Corollary</u>: (Behrstock-Hagen-Sisto) Graph braid groups are hierarchically hyperbolic groups.

<u>Hierarchical hyperbolicity</u>:
- Generalisation of hyperbolicity
- Developed by Behrstock-Hagen-Sisto
- Axiomatises Masur-Minsky's treatment of mapping class groups

<u>Allows us to characterise</u>:

- Hyperbolicity (Behrstock-Hagen-Sisto)
- Acylindrical hyperbolicity (Behrstock-Hagen-Sisto)
- Relative hyperbolicity (Russell)

By developing an explicit structure for graph braid groups, this allows me to recover two theorems of Genevois:

Theorem: (Genevois) $B_n(\Gamma)$ is hyperbolic iff either:

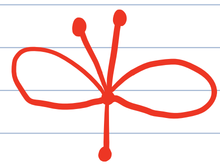
New proof (B.)

i) $n=1$.

ii) $n=2$ and Γ does not contain two disjoint cycles.

iii) $n=3$ and Γ does not contain two disjoint cycles nor a disjoint star and cycle.

iv) $n \geq 4$ and Γ does not contain two disjoint cycles/stars



Theorem: (Genevois) $B_n(\Gamma)$ is either cyclic or acylindrically hyperbolic.

New proof (B.)

Theorem/Conjecture: (B.) We have a similar characterisation of relative hyperbolicity.